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A geometric test for possible Andersonian stress using striated faults

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Abstract

A graphical compilation of the rake guides of fault striations of known sense facilitates a simple test for their compatibility with Andersonian stress conditions. A compilation of negative double strike angles, on tracing paper, is superimposed on the rake guide plot. If the two register, the candidate generating stress is given by the relative position and orientation of the two plots.

Keywords: Shear stress; Tectonic regime; Stress regime; Fault slip

1. Introduction: subject and scope

This paper describes a geometric test of whether a set of striated faults is compatible with a single generating stress state with a vertical principal axis (i.e. 'Andersonian' stress). If the outcome is affirmative, the result provides an estimate of the reduced stress tensor.

This test relies on the assumption that each striation indicates the direction of resolved shear stress on its plane.

The procedure involves trigonometrical calculation, such as might be undertaken using a simple electronic calculator, but otherwise is achieved using only pencil, protractor, ruler and tracing paper.

All conventions and symbols used in this paper accord with Fry (2003).

2. Test procedure and example

The following information is used for each fault (Fig. 1a): the *strike* of the fault plane (angle s, measured clockwise from north); the *dip* of the fault plane (angle δ); the *pitch/rake* of the striation (angle λ); and the movement *sense*. Sense is incorporated by using a 360° range of possible rake angle, giving the direction of the hanging wall relative to the footwall, measured clockwise as viewed from above, from a $\lambda = 0$ reference direction of pure sinistral strike slip.

The stages in the procedure are as follows:

1. Calculation of two new angles for each fault, as shown in Table 1.

   These are $2s$, twice the measured strike, and b, the 'rake guide angle' (Fry, 2003). Both have a range of 360°. In this paper, the ranges are both 0°–360°, but this is inconsequential as it is only the directions indicated by these angles that are used further. The strike polarity—whether the measured angle is s or $(180° + s)$—is also of no significance because the angle is doubled. The rake guide angle projects perpendicularly onto the fault plane as the angle of rake (Fig. 1a). Its value can be calculated using the formula:

$$b = \arctan(\tan\lambda/\cos\delta) \quad (1)$$

   but the result may require addition of 180° to ensure that the direction indicated lies in the same quadrant as that of the measured rake, λ. For this purpose, it is easy to check that b and λ lie in the same 90° range. In Table 1, for example, faults C–E all have $270° < \lambda \leq 360°$, and therefore also $270° < b \leq 360°$, while for faults A, B and F, $180° < \lambda \leq 270°$, so also $180° < b \leq 270°$. (The earlier definition of angle b (Fry 1992b) was not specific enough in this regard for this purpose.) The rake guide angle, b, could also be evaluated graphically.

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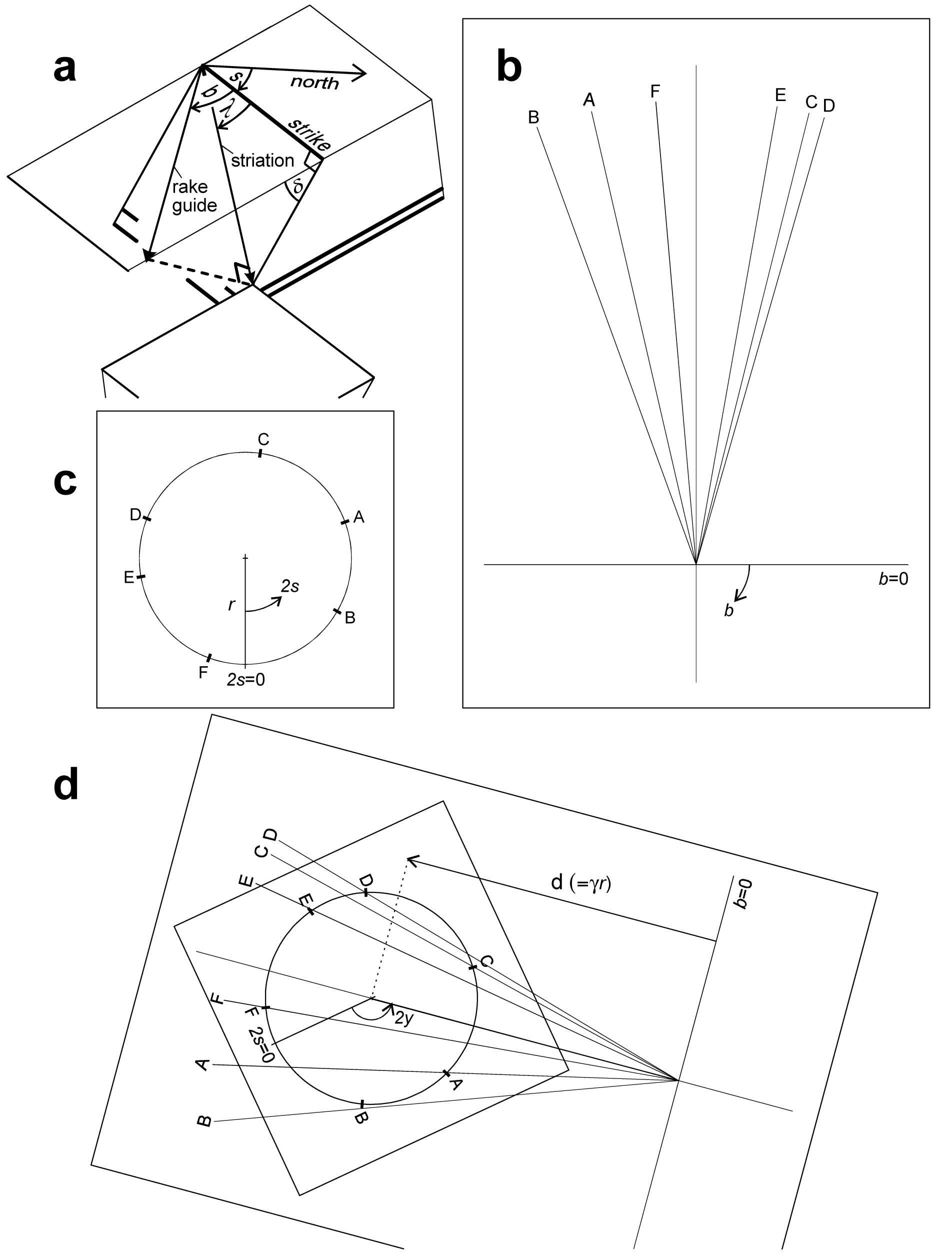

Fig. 1. (a) Angles and directions for a striated fault, in the usage of Fry (2003). The north azimuth, the strike of the fault plane, and the intervening angle, *s*, lie in the horizontal. Strike, the striation direction, and the intervening angle of rake, λ, lie in the fault plane. The rake guide is the horizontal direction, which projects perpendicularly onto the fault plane as the striation direction, with rake guide angle, *b*, projecting to rake, λ. See text for further details. (b) Rake guides are compiled radiating from the intersection of horizontal and vertical axes. In this example, using the values in Table 1, all striations have reverse sense, so all rake guides are upwards from the origin, with rake guide angles, *b*, in the range 180°–360°. (c) The double strike angles, 2*s* in Table 1, are plotted anticlockwise on a piece of tracing paper by marking their intersection with a circle. The radius, *r*, is chosen to be slightly less than the spread of the rake guides in (b). (d) The tracing sheet is superimposed on the plain paper such that each 2*s* point overlies the rake guide of the same fault, while the centre of the 2*s* circle lies on the vertical rake guide axis. The angle that the rake guide plot lies clockwise of the tracing plot is 2*y*—twice the bearing, *y*, of the direction of maximum horizontal compression. The upwards distance, *d*, on the rake guide plot, from the origin to the centre of the 2*s* circle, is γr, where *r* is the radius and γ is the tectonic regime parameter (see text). In this example, $2y = 140°$ and $d\ (=\gamma r) = 3r$. So, the stress state had a maximum horizontal compression lying in the direction 070° and a tectonic regime parameter of 3, according with the alternative specifications, that $R = -1$ (Simón-Gómez, 1986) and $F = -60°$, illustrated for this data set in Fig. 1 of Fry (1992b).

2. Construction of rake guides on a large sheet of plain paper (Fig. 1b).

   Draw horizontal and vertical axes intersecting at a central origin. The rightwards axis is the common reference direction, $b = 0$. For each fault, draw and label the direction of its rake guide, which is the line, outwards from the origin, at angle *b* clockwise from the $b = 0$ reference axis.

3. Construction of negative strike double angles on a sheet of tracing paper (Fig. 1c).

Table 1
Synthetic data set of Table 1 and Fig. 1 of Fry (1992b). All values are in degrees and are given in the range 0°–360°. The data are presented in dip order, to show more easily the increasing divergence between b and λ with increasing dip. Values of rake, λ, and rake guide angle, b, have been redefined to incorporate shear sense

Fault identity		Strike	Dip	Rake (pitch)	Values for use in the test	
New	1992b	s	δ	λ	b	$2s$
A	5	235	41	253	257	110
B	1	30	42	244	250	60
C	2	86	56	294	284	172
D	3	124	60	300	286	248
E	6	320	76	306	280	280
F	4	170	84	230	265	340

Mark a central point. Draw a circle about this origin, and a downwards radius to be used as the $2s = 0$ reference direction. For each fault, mark and label the point on the circle which lies at angle $2s$, measured *anticlockwise* around the circle from the $2s = 0$ reference axis.

4. Attempt to put the two plots in register (Fig. 1d).
   Place the tracing paper over the plain paper and, by trial and error, find the relative position and orientation such that every point on the tracing sheet lies over its own line on the underlying paper. (1) The central point of the circle must lie on the line of the vertical axis. (2) The point representing each fault must, to be compatible with Andersonian stress conditions, lie on the line that represents the same fault.

3. Interpretation: case of perfect register of points with their lines

3.1. All data in register

If every point registers with its own line at the same time, as in Fig. 1d:

1. The entire set of faults recorded is compatible with the same Andersonian stress conditions.
2. The downwards axis on the plane paper lies at angle $2y$ from the zero axis on the tracing sheet, where y (in the usage of Fry (1992b) after Simón-Gómez (1986)) is the bearing of the direction of maximum horizontal compression.
3. The distance, d, upwards (on the plain paper) from the origin of the rake guides to the centre of the circle (tracing) is a proportion of the circle's radius, r, equating to the 'tectonic regime parameter':

$$d/r = \gamma = (\sigma_{h1} + \sigma_{h2} - 2\sigma_v)/(\sigma_{h1} - \sigma_{h2}), \quad (2)$$

where subscript v indicates vertical, h horizontal and $\sigma_{h1} \geq \sigma_{h2}$ (Célérier, 1995). From this value, any other desired formulation of stress ratio may be calculated.

3.2. Subsets of data in register

If only a subset of the fault points overlie their lines simultaneously, the subset is compatible with a single generating stress state, which can be evaluated as above. Theoretically, any number of such subsets may be independently identified, each with its own relative position and orientation of tracing and plain paper indicating its stress state.

4. Errors, uncertainties and approximations

In practice, it is likely that the match between points and their lines will be only approximate, in which case appreciation of the acceptability and likely magnitudes of errors may be required. Errors differ in type and likely magnitude for different ranges of dip and rake values. Consequently, some points should be weighted more heavily than others in the fitting procedure, and some, perhaps, discarded altogether.

4.1. Unacceptable uncertainty at dips less than 12°

For Andersonian stress conditions, as a plane tends to horizontal, the shear stress on it should tend to zero, and fault movement should not happen. Striated faults of low dip are not likely to be compatible. It is unlikely that they could contribute to definition of any stress state that is Andersonian.

Only slight divergence of the principal stress axis from verticality can result in large modifications of rakes of resolved shear stress on shallowly dipping planes. Consequently, inclusion of such data may detract from an otherwise good stress approximation given by planes of greater dip.

As the dip of a plane decreases, its strike becomes increasingly ill determined until, for a horizontal plane, strike is indeterminate. At small dip, the recorded value of strike is extremely sensitive to any misjudgement of the horizontal.

For all the above reasons, *it is suggested here that fault planes with a dip of less than 12° be excluded.* Such data cannot contribute reliably to the test described in this paper, or to any other method that assumes ('Andersonian') that a plane's strike lies in a principal stress plane. Fry (1992b) was wrong to extend to low dip the claim for his $(F,2y)$ method (p. 1128) that "departure from the assumed verticality of one principal axis leads to $<10°$ error in $(F,2y)$" (Célérier, pers. commun.).

This 12° threshold is set arbitrarily. At 12°, if there is error in field measurement such that a line of plunge 1° is taken as strike, the error induced in $2s$ is not more than 10°. For a 2° error in the horizontal, a threshold dip of 22° would be required to keep error in $2s$ below 10°. One may wish to vary the threshold dip to be applied, to match the possible magnitude of errors from this source to the uncertainty accepted from the rest of the data, as discussed in Section 4.3, below.

As pointed out elsewhere (Fry, 1992a), although strike and rake are ill determined for planes of low dip, the directions specified by their combinations are not (and are better measured as azimuth and plunge). Therefore, an appropriate method for data, that include striated faults of low dip will not only be one which determines *whether* a principal stress axis is close to vertical, rather than making this assumption, but will do so by using, for the purpose of minimising residuals, reconstitutions of any poorly determined raw data into well determined measures, as does Fry's (1999) use of direction cosines.

4.2. Unacceptable uncertainty in b at low rake on steep faults

The main source of uncertainty in the value of b arises from magnification of measurement errors, by the projection from the fault plane to the horizontal, in accord with Eq. (1). This magnification becomes a serious consideration for striations with rake within 30° of horizontal on planes dipping more than 60°. For the λ range of 0°–360° used in this paper, this translates for sinistral displacement to $\lambda < 30°$ or $\lambda > 30°$ (within 30° of $\lambda = 0°$ or $\lambda = 360°$), and for dextral displacement to $150° < \lambda < 210°$ (within 30° of $\lambda = 180°$). To avoid such lengthy specifications, in the discussions that follow all such cases will be assumed covered by the phrase 'rake of less than 30°' and by the formulation '$\lambda < 30°$', with similar usage extended to other angles of rake, such as 15°.

At the limits to this range ($\delta = 60°, \lambda < 30°$ or $\delta > 60°, \lambda = 30°$), magnification is approximately by a factor of two, but it increases to a factor of four or more at dip of 75° or rake of 15°. At combinations of dip greater than 75° and rake less than 15°, this increase is more rapid. *It is suggested here than data should be excluded if* $\delta > 75°$ *and* $\lambda < 15°$. Whether to use other data within the range ($\delta > 60°, \lambda < 30°$) is a question to be addressed in light of the accuracy achieved by other data, as discussed in Section 4.3.

As with shallow dip data, this exclusion of data should not be considered a failing of this particular method. As a plane tends to vertical, the Andersonian symmetry requires that the direction of resolved shear stress tend to horizontal, as illustrated by Fig. 13 of Célérier (1995) and by Fig. 6 of Fry (2003). So, nearly horizontal shear on steep planes can never provide a good constraint on possible values of γ by any method, although the constraint it provides on possible orientations of the horizontal principal stress axes may be valuable.

4.3. Errors for data outside the ranges addressed above

In the previous two sections, the approach taken to uncertainty is based on comparability. It is assumed that the measurement errors in the three measured angles, s, δ and λ, are equivalent. The error in measurement of strike is magnified by a factor of two in plotting $2s$. Therefore, for practical purposes, it is appropriate to accept error of similar absolute magnitude in b, as one cannot tell, from a mismatch in this test, which may have contributed. The limits to the ranges specified in Sections 4.1 and 4.2, above, are close to those at which measurement errors are magnified by a factor of two. For the entire range of dip and rake combinations outside those ranges, the magnification factor of measurement errors is about two or less. Therefore, a reasonable practical approach for most data is to consider, as a perfect match, all points that lie closer to their corresponding lines than an angular error in either b or $2s$ of twice that of normal angular measurement.

Mismatches of somewhat more than twice measurement error may be treated as possible matches, if they derive from data at the limits of those ranges discussed in Sections 4.1 and 4.2. If such mismatches come from data well outside those ranges, they indicate only an imperfect approximation to the assumption of a single and Andersonian stress.

5. Conclusions regarding status of data and interpretation of mismatch

From the above consideration of errors and uncertainties, it is concluded that:

1. If dip is less than 12°, the fault should not be included in the test. The stress that generated it was probably not Andersonian.
2. If the dip is greater than 75° and the rake is less than 15°, the fault should not be included within this test. Its generating stress was possibly Andersonian, and it may provide useful constraints on horizontal stress orientation, but it cannot contribute to determination of γ.
3. Faults with dip greater than 60° and rake within 30° of horizontal, not covered by point 2, above, may be acceptable at mismatches greater than double the normal measurement error, but should not be given preference in determining the fit between the strike and the rake guide plots.
4. For each fault ouside the ranges in points 1–3, above, mismatches in excess of twice the possible error of the original measurements indicate either, if small, that the assumption of movement in response to a single Andersonian stress state is only an approximation, or, if large, incompatibility with the Andersonian stress indicated by the relative position and orientation of the two plots.

6. Conclusions and perspective on the usefulness of the method

6.1. Overview of context: methods and assumptions

There are two approaches to testing fault/slip data for compatibility with an Andersonian stress. One is to undertake a stress determination with essentially only one assumption—that if the data can be explained as the directions of maximum shear stress imposed by a single stress tensor, then they should be—and see if it gives a principal stress axis that is vertical. There are many published methods for such determinations, which will not be discussed further here. They tackle a problem which, viewed geometrically, lies on a curved four-dimensional surface within an, at best, five-dimensional parameter space (Fry, 1999), which requires substantial computation.

This method belongs to the alternative approach, of compiling the data assuming a vertical principal stress axis. This is much simpler, corresponding geometrically to a problem on a two-dimensional surface within three-dimensional parameter space, susceptible to graphical analysis, either on a stereogram (Fry, 1992b) or in some projection (Simón-Gómez, 1986; Célérier and Séranne, 2001; this paper). If data plot in such a way as to indicate compatibility with a single stress tensor, we draw two conclusions. The first is that the data should be explained by a single stress tensor. The second is that that tensor is Andersonian, with a vertical principal axis. The latter is justified on condition of the former. Unfortunately, the former conclusion, that the data are compatible with a single stress tensor at all, may not be justifiable. The data of Table 3 of Fry (1992b) appear to give a good match to an Andersonian stress tensor (Fig. 10 of Fry (1992b)) but do not give good agreement to a single tensor when the Andersonian assumption is removed. This issue deserves further investigation.

6.2. Overview of graphical methods

Of those that assume a vertical principal stress axis, different methods are appropriate to different purposes.

The test described in this paper may be mainly of didactic use, applied to small sets of data. Its two elements—rake guide angles and double strike angles—are conceptually only slightly removed from actual rake and strike, and the method assists understanding of the relationship between the two. However, when used for a large data set, the arrays of lines on both constituent plots become crowded, making the matching procedure difficult. With heterogeneous data, it is almost impossible to know whether one has tried all possible matches of position and orientation that might identify homogeneous subsets.

For small sets of natural data, the easiest graphical test of compatibility with the Andersonian assumption, which also gives an estimate of the reduced stress tensor, is the half great circle $F(R)$,$2y$ stereogram of Fry (1992b, Figs. 9 and 10). This also has some didactic use regarding symmetry constraints and likely misinterpretations of data. It is also a method that can be used when some data (plotted as full great circles as in Fig. 1b of Fry (1992b)) are of unknown shear sense.

The historically pioneering (y,R) projection of Simón-Gómez (1986) is more work and more potentially misleading than the otherwise equivalent alternative of the (F,$2y$) stereographic projection (Fry, 1992b).

For larger data sets, point plots of data are more intelligible than intersecting line plots. If the data accord with one or two Andersonian stress states, the pole version of the (F,$2y$) stereogram of Fry (1992b) gives easy rapid confirmation. However, if they do not, the amount of information that may be retrieved from the plot is usually minimal. This method also requires that the sign of the tensor, given by shear senses, must be checked separately.

For complex data sets, the method of Célérier and Séranne (2001) has the major advantage that it is the only method using a plot that retains the original strike and rake information. This is even more informative if the symbols carry some information about the dips of fault planes. The plot remains valuable, even if the Breddin's graph has not succeeded in providing a reasonable stress tensor for some or all the data. The Breddin's graph is placed over the data as a moveable overlay—hardly an onerous procedure, if the plot has previously been produced, as valuable in its own right. The other methods mentioned above produce plots that turn out to be of little use if a simple stress determination fails.

Making a choice between these possible methods may be unwise. A quick test may be productive in simple cases. It may be that, with increasingly complex data, it is increasingly unwise to build in the Andersonian assumption of vertical principal stress. It depends on situation and purpose.

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